Tau polarization in charge current neutrino-nucleon Deep Inelastic Scattering

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Numerical results for the degree of polarization of τ^- produced in (CC) neutrino - nucleon Deep Inelastic Scattering (DIS) are presented. Calculations are done in the threshold region, where the τ^- scattered by the small angles can be partially polarized. The cross sections and polarization are calculated by using the GRV98 parton distributions functions (PDF's) and the GRV98 with modifications of A. Bodek at.al. [9].

1. Introduction

The oscillation of muon to tau neutrinos is the most acceptable solution of the atmospheric muon neutrino deficit measured in the Super Kamionkande. Experiments such as ICARUS, OPERA will verify it by observation of tau neutrinos resulting from oscillations of the CNGS muon neutrinos. The expected number of ν_τ events will not be large. Therefore a detailed analysis of experimental data based on a good theoretical description of the neutrino-matter interaction will be necessary.

The tau leptons produced in the neutrino-matter interaction can not be measured directly because of their short lifetime, therefore their decay products must be detected. The large mass of the tau lepton implies that it may not be fully polarized [1,2]. The degree of its polarization is one of the parameters of its decay [3]. Hence it is important to take into consideration the polarization properties of τ. It can play an important role in the analysis of experimental data.

The polarization of the tau lepton produced in the neutrino - matter interaction has been the subject of several papers [4,5,6,7]. In [4,5] the calculation were performed for quasielastic neutrino-nucleon scattering, for single pion production, and for deep inelastic scattering. The polarization vector was obtained from the spin density matrix.

In the CNGS experiments the majority of neutrinos is detected by the observations of the products of their inelastic scattering on nucleons. Usually to describe the events with small values of hadronic invariant mass some kind of the resonance model like Rein-Sighal is used. Other events are described within the Deep Inelastic Scattering formalism.

The tau leptons produced in the inelastic scattering are usually characterized by higher degree of polarization [4] but for small neutrino energies and small scattering angles the taus may be partially polarized. In the present paper we focus on this energy and scattering angle values.

The DIS formalism is described by the structure functions expressed in terms of parton distribution functions (PDF's). We adopt the GRV98 PDF's from [8], but it seems that in the region of small Q^2 they need to be modified. A. Bodek et. al. [9] proposed a modification based on the new experimental data from JLab. We discuss how these modifications influence the produced leptons degree of polarization and its energy distribution.

2. Theoretical description

We consider the following process:

$$\nu(k) + N(p) \rightarrow \tau^-(k', s^\mu) + X$$

The produced tau lepton is polarized and is characterized by a spin four-vector s_μ which satisfies in any frame the relations: $k'_\mu s^\mu = 0$, $s_\mu^2 = -1$. The cross section is proportional to the contraction of the lepton tensor with the hadron tensor:

$$d\sigma(k, q, s) \sim L_{\mu\nu} W^{\mu\nu}.$$

We use the well known form of the hadron tensor [10]

$$
\begin{aligned}
W_{\mu\nu} &= -g_{\mu\nu} F_1 + \frac{p_\tau p_\nu}{p \cdot q} F_2 - \mathrm{i}\frac{\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta}{2 p \cdot q} F_3 \\
&+ \frac{q_\mu q_\nu}{p \cdot q} F_4 + \frac{p_\mu q_\nu + p_\nu q_\mu}{2 p \cdot q} F_5
\end{aligned}
\tag{1}
$$

F_i (i=1,2,...,5) are the structure functions. The lepton tensor:

$$
\begin{aligned}
L_{\mu\nu} &= 8\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k'_\alpha k^\alpha - \mathrm{i}\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta\right) \\
&+ 8m s^\alpha \left(k_\nu g_{\alpha\mu} + k_\mu g_{\nu\alpha} - g_{\mu\nu} k_\alpha - \mathrm{i}\epsilon_{\mu\nu\beta\alpha} k^\beta\right)
\end{aligned}
$$

is a sum of two contributions: one is linear in the tau lepton momentum k' the other in its mass m and its spin four-vector s^μ.

The polarization of the τ measured in the direction of the four-vector s_μ is given by [11]:

$$\mathcal{P}_{s_\mu} = \frac{d\sigma(k, q, s) - d\sigma(k, q, -s)}{d\sigma(k, q, s) + d\sigma(k, q, -s)} \equiv P_\mu s^\mu \tag{2}$$

which defines P_μ – the polarization vector of the tau.

We introduce the four-vectors e_l^μ, e_t^μ, e_p^μ [1,7] which in the LAB frame have the following form:

$$e_l^\mu = \frac{1}{m}\left(|\mathbf{k}'|, E_\tau \frac{\mathbf{k}'}{|\mathbf{k}'|}\right), \quad e_t^\mu = \left(0, \frac{\mathbf{k} \times \mathbf{k}'}{|\mathbf{k} \times \mathbf{k}'|}\right),$$

$$e_p^\mu = \left(0, \frac{\mathbf{e}_t \times \mathbf{k}'}{|\mathbf{k}'|}\right).$$

Writing the polarization four-vector as a linear combination of k', e_l, e_p and e_t:

$$P^\mu = \alpha k'^\mu + e_l^\mu \mathcal{P}_l + e_p^\mu \mathcal{P}_p + e_t^\mu \mathcal{P}_t \tag{3}$$

defines its longitudinal $\mathcal{P}_l$, perpendicular $\mathcal{P}_p$ and transverse $\mathcal{P}_t$ components.

To define the degree of polarization it is useful to go into the rest frame of the tau lepton where the spin four-vector s_μ has the form:

$$s_\mu = (0, \hat{\mathbf{s}}), \quad \hat{\mathbf{s}}^2 = 1$$

Thus the polarization measured in the direction given by $\hat{\mathbf{s}}$ is equal to:

$$\mathcal{P}_\mathbf{s} = -\mathbf{P} \cdot \mathbf{s} = -|\mathbf{P}| \cos(\beta)$$

β being the angle between $\mathbf{P}$ and $\hat{\mathbf{s}}$. It is easy to notice that in this frame $\mathbf{k}' = 0$ and (3) implies:

$$\mathbf{P} = \mathbf{e}_l \mathcal{P}_l + \mathbf{e}_p \mathcal{P}_p + \mathbf{e}_t \mathcal{P}_t \tag{4}$$

The quantity

$$\mathcal{P} = |\mathbf{P}| = \sqrt{\mathcal{P}_l^2 + \mathcal{P}_p^2 + \mathcal{P}_t^2}. \tag{5}$$

is called the degree of polarization of tau and is frame independent.

In the LAB frame we choose the coordinate system in which $\mathcal{P}_t$ vanishes and we obtain the following analytic formulas for the differential cross section and polarization vector components [4]:

$$
\begin{aligned}
\frac{d\sigma}{dE_\tau d\cos(\theta)} &= \frac{G^2 p_\tau}{4\pi M} F \\
\frac{G^2 p_\tau}{4\pi M} &\left(2F_1(E_\tau - p_\tau \cos(\theta)) + F_2 \frac{M}{q_0}(E_\tau + p_\tau \cos(\theta))\right. \\
&\left.+ \frac{F_3}{q_0}\left(E E_\tau + p_\tau^2 - (E + E_\tau) p_\tau \cos(\theta)\right) - \frac{m^2}{q_0} F_5\right)
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
\mathcal{P}_p &= \\
-\frac{1}{2} &\left(2F_1(p_\tau - E_\tau \cos(\theta)) + F_2 \frac{M}{q_0}(p_\tau + E_\tau \cos(\theta))\right. \\
&\left.+ \frac{F_3}{q_0}((E + E_\tau) p_\tau - (E E_\tau + p_\tau^2)\cos(\theta)) - \frac{m^2}{q_0} F_5 \cos(\theta)\right)/F \\
\mathcal{P}_p &= -\frac{m \sin(\theta)}{2}\left(2F_1 - F_2 \frac{M}{q_0} + \frac{E}{q_0} F_3 + \frac{E}{q_0} F_5\right)/F
\end{aligned}
$$

where E is the neutrino energy, E_τ denotes tau energy, $q_0 = E - E_\tau$ is the energy transfer, p_τ stands for the lepton momentum and M for nucleon mass. It is assumed that $F_4 = 0$ and $x F_5 = F_2$.

3. Numerical results

The allowed kinematical region for the DIS formalism is restricted by the condition for minimal possible hadronic invariant mass $W > M + m_\pi$.

The results for the GRV98 (LO) PDF's are obtained with a freeze of Q^2 at 0.8 GeV^2. The implementation of the GRV98 with modifications of A. Bodek is based on the description from the original paper [9].

In the figures 1 and 2 we present the dependence of the degree of the polarization of τ^- on
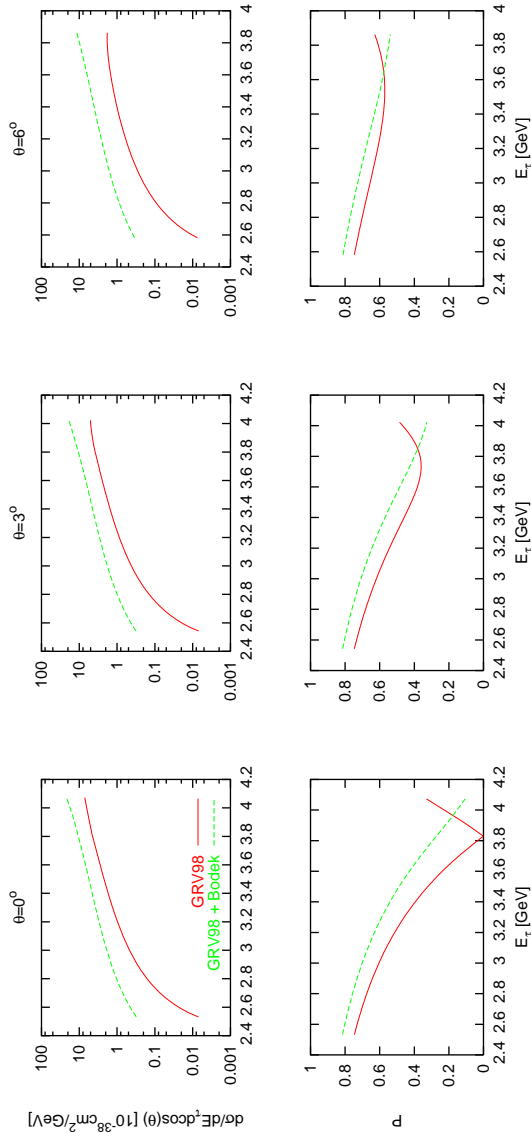
Fig. 1. In the charts the dependence of the degree of polarization of τ^- on its energy and correspond and the differential cross section are presented. The calculations are done for the three scattering angles $\theta = 0^o$, 3°, 6°. We compare the plots in the first row where the differential cross sections are shown with the second row with appropriate plots of the degree of polarizations. The calculations are done for the neutrino energy of 4.5 GeV and two sets of PDF's GRV98 – solid (red) line and GRV98 with A. Bodek at al. modifications – dashed (green) line.

E_τ energy. The tau lepton is produced in neutrino - isoscalar target $(N = \frac{1}{2}(p + n))$ scattering. The calculations are done for 4.5 and 8 GeV neutrino energy and for three scattering angles $\theta = 0^o$, 3°, 6°. The charts of the degree of polarization are presented together with the plots of the corresponding differential cross sections given by formula (6).

For the neutrino energy of 4.5 GeV (Fig. 1) the cross sections which are obtained by using the GRV98 PDF's with modifications are several times bigger than those calculated by adopting the GRV98 PDF's. However the degree of polarization are comparable.

For the GRV98 PDF's the degree of the polarization has minimum which divides the polarization curve into two branches. These branches correspond two different sings of longitudinal polarization and at the minimum $\mathcal{P}_l$ vanishes. The lower (higher) energies of τ^- correspond to positive (negative) sign of $\mathcal{P}_l$ respectively. This effect was clearly explained by K. Hagiwara at. al. [4]. In the center of mass frame (CM) all tau leptons produced in νN reaction are left-handed and are scattered in the all directions. Performing the Lorentz boost to the LAB frame can transform a left-handed τ^- scattered in backward direction in the CM frame into a right-handed one scattered in the forward directions in the LAB frame. The degree of polarization of τ^- calculated by using GRV98 PDF's with modifications have only the first branch of polarization curves. It means that for the neutrino energy of 4.5 GeV the leptons are characterized by positive sign of the helicity.

In the case of the neutrino energy of 8 GeV Fig. 2, the mentioned above effect appear for the both used set of PDF's. The cross sections as well as the degree of polarization of τ^- calculated by using the GRV98 PDF's with and without modifications are almost the same.

Summary

The degree of polarization of the τ^- produced in the νN inelastic scattering in the threshold region for the small scattering angles has the minimum in the same place where the differential cross section reaches its maximum. The τ^- scattered forward which have energy about close to the neu-
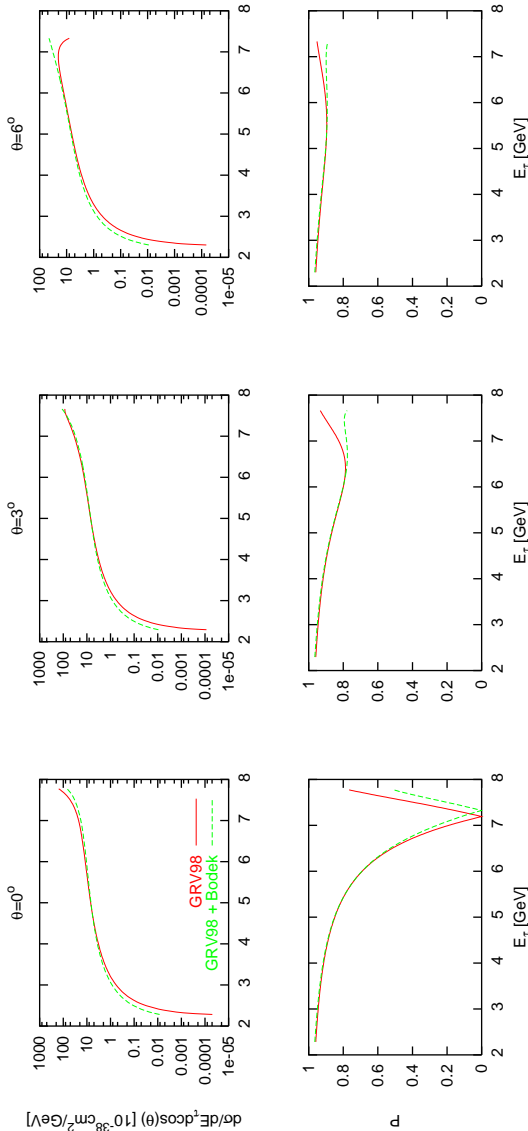
trino energy will be only partially polarized.

In the threshold region the PDF's the GRV98 with and without corrections give very different values of cross section. The $\mathcal{P}_l$ obtained by using GRV98 with modifications (for E=4.5) has positive sign. The differences of results given by application of two investigated sets of PDF's disappear for higher neutrino energies.

Summary

The author would like to thank J. Sobczyk and C. Juszczak for reading of the manuscript and interesting comments.

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Fig. 2. In the charts the dependence of the degree of polarization of τ^- on its energy and correspond and the differential cross section are presented. The calculations are done for the three scattering angles $\theta = 0^o$, 3°, 6°. We compare the plots in the first row where the differential cross sections are shown with the second row with appropriate plots of the degree of polarizations. The calculations are done for the neutrino energy of 8 GeV and two sets of PDF's GRV98 – solid (red) line and GRV98 with A. Bodek at al. modifications – dashed (green) line.